Description of Structures in Terms of Polyhedrapacking

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The structures of alloys can be described in terms of polyhedrapacking. We studied *all* structure types found in the binary systems T^5 or T^6 with \hat{B}^3 or \hat{B}^4 (T^5 : V, Nb, Ta; T^6 : Cr, Mo, W; B^3 : Al, Ga, In, Tl; B^4 : Si, Ge, Sn, Pb) elements. Most of the structures examined until now could be built up with one or two polyhedra, only in a few cases more than two polyhedra are required. It is found that there are two types: a three-dimensional distribution of discrete polyhedra sharing corners, edges or faces and a layer-like distribution. This model proved valid for all structure types studied. Classification of the structures according to their polyhedrapacking criteria is introduced (Table 1). Table 2 includes the coordination numbers of all atoms in the studied structures.

(Keywords: Coordination polyhedra; Polyhedrapacking model," Classification of structures; Systematics of crystal structures)

Beschreibung yon Strukturen mittels des Polyederpackungs-Modells

Die Strukturen von Legierungen können mit dem Polyederpackungsmodell beschrieben werden, dessen Gfiltigkeit bewiesen wurde. Wir haben *alle* Strukturtypen, die in den binären Systemen T^5 oder T^6 mit B^3 oder B^4 Elementen auftreten, beschrieben. Es zeigt sich, dab die meisten Strukturen mit einer oder zwei Polyedersorten aufgebaut werden können. Eine Klassifizierung der Strukturen gemäß ihrer Polyederpackungstypen wurde eingeführt (Tabelle 1). Zwei Typen von Polyederaufbau wurden festgestellt: entweder ein 3-dimensionales Geriist aus einzelnen Polyedern, die Ecken, Kanten oder Flächen miteinander gemeinsam haben oder Polyederschichten. Tabelle 2 enthält die Koordinationszahlen aller Atome der behandelten Strukturtypen.

Introduction

A fruitful approach to understand the crystal chemistry of alloy structures is to consider the coordination spheres. Further the search for a model to describe these "sometimes" complicated structures led us to the polyhedrapacking principle. The aim is to present a simple description of the complex alloy structures and to systematize the different known structure types.

Coordination Polyhedra

To define a coordination polyhedron, we first limit the coordination sphere. *Girgis*¹ introduced the $\sum (r_A + r_B)/d_{AB}$ -method for this purpose. $\sum_{i=1}^{n} (r_A + r_B)$ is the sum of the radii of the atoms concerned *(Teatum,* \overline{G} *schneidner, Waber*²) and d_{AB} is the interatomic distance between these atoms A and B. The $(r_A + r_B)/d_{AB}$ versus the number of atoms are represented in a histogram. The widest range in which no atoms are present is called the "maximum gap" *(Brunner3).* 95% of the studied cases show such a "maximum gap" *(Bhandary, Girgis^{4,5})*. The number of nearest neighbours before the "max. gap" is taken to be the conventional coordination number (CCN). These atoms (neighbours) constitute the coordination polyhedron for the corresponding (central) atom *(Girgis¹)*. An example is shown in Fig. 1.

Fig. 1. Distribution of near neighbours as a function of their $(r_A + r_B)/d_{AB}$ for Cr in the $Cr₃Si$ structure type

The Principle of Describing Structures with Coordination Polyhedra as Building Blocks

Procedure

The coordination polyhedra of all the atomic positions of the structure will be determined with the help of the "max. gap" method as mentioned above.

The structure will be described by packing the least number of polyhedra types (one or two are in most cases sufficient). *All the atoms* in the unit cell *must be* included in the structure building polyhedra. The polyhedra should not penetrate each other.

If there is more than one possibility to describe the structure, the following priorities are considered:

1. The polyhedron (polyhedra) of the transition metals is considered first.

2. The least number of polyhedra types are considered.

2.1. Known polyhedra are considered first, e.g. *Kasper* polyhedra (CN12, 14, 15, 16), cubooctahedron, tetrakishexahedron, rhombohedron, cube, trigonal prisms etc.

2.2. Polyhedra with higher symmetry (higher order).

2.3. Polyhedron with the clear "max. gap".

2.4. Polyhedron with the lower CN.

Validity of this Model

The polyhedrapacking principle proved its validity for all structure types treated (a part of them in Table 1). More details are found in $Refs.$ ⁵⁻⁸. Therefore it is expected that this principle is valid for all intermetallic structures.

Some Examples

We would like to show the steps to describe a structure with polyhedrapacking in the case of $V_5A\mathbf{l}_8$ as a representative for the y-brass structure type ($Cu₅Zn₈$) and of CrAl_{6.6} as a representative for the α -V₇Al₄₅ structure type.

 $Cu₅Zn₈$ (D 8₂) Structure Type: Representative: $V₅Al₈$

Crystallographic data:

 V_5Al_8 Cu₅Zn₈ type c₁₅₂ I₄₃m a = 9.207Å $Z = 4$

There are two crystallographically independent \bf{y} atoms, \bf{V}_1 and \bf{V}_2 , in the asymmetric unit. V_1 has CN 12 and V_2 13. The polyhedron around V_1 is an icosahedron having 3 m (C_{3v}) symmetry. The 13-verticon around V₂ has mm $2(C_{2v})$ symmetry and is bounded by 22 triangular faces, see Fig. 2.

The structure can be built up by packing V_1 polyhedra. Two V_1 polyhedra share a triangular face along [110].

Another two V_1 double icosahedra in a higher plane share with the lower two V_1 polyhedra 4 triangular faces which are shaded. These 4 icosahedra form a "tetrahedron". These tetrahedron units "centres" are in 000 and 1/2 1/2 1/2. Each tetrahedron unit shares with the next higher tetrahedron unit the shaded atoms in Fig. 3.

VsA___~

cI 52 Cu₅Zn₈ type

Fig. 3. Description of the Cu₃Zn₈ structure type (representative: V_5A_8) in terms of polyhedrapacking. Shared atoms or faces are shaded. Numbers are the coordinates in the z-direction

 $\begin{array}{c} 5 & \overline{8} \\ 0 & \overline{0} \end{array}$

 $\hat{\boldsymbol{\beta}}$

 Cr Al 6.6

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Table 1

A : STRUCTURES DESCRIBED BY PACKING OF ONE POLYHEDRON

Type of packing:

I : 3-dimensional arrangement of discrete polyhedra sharing corners, edges or faces

I[: a layer like distribution of polyhedra (sheet polyhedrapacking)

Table 1 *(continued)*

B : STRUCTURES DESCRIBED BY PACKING OF TWO POLYHEDRA

Type of packing:

I : 3-dimensional arrangement of discrete polyhedra sharing corners, edges of faces

11 : a layer-like distribution of polyhedra(sheet polyhedrapacking)

C : STRUOTURES DESCRIBED BY .PACKING OF MORE THAN TWO POLYHEDRA

Type of packing:

I : 3-dimensional arrangement of discrete polyhedra sharing corners, edges or faces

II : a layer-like distribution of polyhedra (sheet polyhedrapacking)

Table 2. Summary of coordination numbers of individual atoms in the treated structures*

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 α -V₇Al₄₅ Structure Type: Representative CrAl_{6.6}

CrAl_{6.6} crystallizes in the monoclinic space group C₂/m with 104 atoms per unit cell. The asymmetric unit contains 3 independent Cr atoms and 16 Al atoms. The Cr atoms $(Cr_0, Cr_1$ and $Cr_2)$ have CN 12, forming distorted icosahedra. Cr₀ polyhedron possesses $2/m$ (C_{2b}) symmetry, Cr₁: $m(C_s)$, Cr_2 : $1(C_1)$ respectively.

One can describe this structure in terms of packing these 3 Cr polyhedra, Fig. 4. Cr_0 polyhedron shares an Al_8 atom with Cr_1 polyhedron which shares two triangular faces with a complex unit of Cr_2 polyhedra (which is two icosahedra sharing a five membered ring of atoms). In the figure the polyhedra and their contacts are shown in the *ab* and *ca* planes. The structure is shown in ca-projection. These units of the three polyhedra repeat themselves in the unit cell, sharing the shaded atoms among each other as shown on the right hand side of the figure. The complex units of Cr_2 share two triangular faces with each other. Along the b axis the Cr_2 units share apices.

Structure Classification According to Polyhedrapacking

A. We classify the structure types as follows (Table 1):

1. a. Structures described by packing of *one polyhedron.*

b. Structures described by packing of *2 polyhedra.*

c. Structures described by packing of *more than 2 polyhedra.*

2. a. 1. a to 1 c will be subdivided according to the *number of vertices of the polyhedron* (i.e. according to CN of the central atom around which the polyhedron considered is built up); beginning from lower coordination numbers going to higher ones.

b. Polyhedra having the *same number of vertices* will be subdivided according to their symmetry. We begin with cubic point group symmetries followed by hexagonal, trigonal, tetragonal, orthorhombic, monoclinic and triclinic point group symmetries.

B. Two types of polyhedrapacking were observed:

1. A three dimensional distribution of discrete polyhedra sharing corners, edges or faces.

2. A layer-like distribution of polyhedra (sheet polyhedrapacking). 2 is in fact a special case of 1.

This further feature is included in Table 1, "Type of Packing" column.

Conclusions and Discussion

1. The polyhedrapacking principle *has proved* valid for all structures treated (Table 1). It is expected that this principle is *valid for all* *intermetallic structure types.* Other concepts are valid just for some structures. At least it has not been studied, if any of these concepts is valid for all (most) structures available.

For example *Samsons* fascinating model⁹ for giant molecules which describes the structures by packing *Friauf* polyhedra is limited by the fact that similar atoms in the structure should be surrounded by *Friauf* polyhedron.

Structures described by "Kagomé Nets" must exhibit these nets in the structure. For more details on structure description models see *Girgis lo.*

2. It delivers a simple, clear representation of the structures.

3. Different relationships are found:

 V_5Si_3 and V_6Si_5 are both built up by similar polyhedra, although they belong to different structure types. V_5Si_3 : W_5Si_3 , ti32, I4/mcm; V_6Si_5 , o I 44, Ibam. V_5Si_3 (W₅Si₃ type) and V_5Ge_3 (Mn₅Si₃ type) have both a *Kasper* polyhedron as a building unit. In the $V₅Ge₃$ it is very distorted, therefore we call it *Kasper-like* polyhedron.

4. Icosahedra, *Kasper, Kasper-like* 14 verticon and rhombic dodecahedron play a big role in the description of these structures, although they belong to different structure types.

Acknowledgments

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